THE NOVEL CFRG -BASED COMPLEX FUZZY TRANSFER LEARNING SYSTEM

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Abstract. Today, the rapid development of the internet has led to a data explosion; the complex fuzzy transfer learning (CFTL) model has received increasing attention from the academic community due to its various real-world applications, such as solar activity, digital signal processing, time series forecasting, etc. CFTL combines Transfer learning and Complex Fuzzy Logic in a framework to solve the problem of learning tasks with no prior direct contextual knowledge, which is stored, retrieved, and organized in the data structure. Data structures are important in computational intelligence because they are key performance indicators for systems or models. Therefore, to improve the performance of the previous CFTL, this paper investigates a novel complex fuzzy decision tree (CFDT) structure to represent the complex fuzzy rules and provides a transfer learning model for a complex fuzzy inference system. In contrast with prior axis-parallel decision trees in which only a single feature or variable is considered at each node, the node of the proposed decision tree structures includes complex fuzzy inference rules that contain multiple elements. Multiple features for each node help minimize the size. To prove the efficiency of the proposed framework, we carry out extension experiments on numerous instances (datasets). Experimental results demonstrate/exhibit that our offered performs better than the prior framework regarding accuracy and the size of the produced trees.

Keywords. Complex fuzzy set, mamdani complex fuzzy inference system, transfer learning, fuzzy transfer learning, complex fuzzy transfer learning, complex fuzzy rule tree.

1. INTRODUCTION

Decision trees are one of the most well-known techniques for deriving categorization rules from data. They are widely applicable for several reasons, making them visible. Firstly, the decision tree's accuracy is better than that of other categorization models [1]. Secondly, the design of the majority of decision trees only necessitates making a few changes to parameters. Thirdly, the resulting categorization models are simple to understand due to their intuitively

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appealing topology. Several authors have published different methods to generate a rule-based decision tree. The fundamental idea is to create a node at each level of the hierarchy for each class, with each node denoting an oblique geometric structure represented by a fuzzy rule. The tree structure significantly improves the rule-based system which is used to characterize uncertain data [2] applied to classification problems in many fields [3, 4, 5, 6].

Samantaray [7] introduced a fuzzy rule foundation with decision tree (DT) initialization for power quality (PQ) event classification. The author also claimed that the DT-fuzzy method yields more accurate results for classifying PQ events compared to a heuristic fuzzy rule-based approach. [8] suggested an intelligent FIS that gives diabetes patients content recommendations using a decision tree rule induction technique to construct the rules for predicting the diabetes diagnostic model. The work [9] presented a decision-tree-based Fuzzy Inference System (FIS) for making optimal choices in developing reduced-order finite element models for complex and nonlinear problems. A novel rule generation and activation method for an extended belief rule-based (EBRB) system based on an improved decision tree is proposed in [10]. [11] introduced a multilayer tree structure (MTS) for a Belief rule-based expert system to handle uncertain problems.

Nowadays, as changes are made to the process (periodicity) of the data, imprecision in our daily lives and the uncertainty of real-life data concurrently emerge. Current theories cannot explain a period of partial data ignorance. They need to be more comprehensive to account for periodic information and contain exact but ambiguous factors, which causes information loss. To cope with periodic elements in data, the Mamdani complex fuzzy inference system (M-CFIS) [12] was recently introduced with a specific inference structure and some extensions such as [13, 14].

The rule base, which may be residual and inconsistent with the dataset, is a drawback of inference systems based on complex fuzzy sets. Recently, [15] presented a complex fuzzy transfer learning (CFTL) to address these restrictions by utilizing a transfer learning technique. The authors developed a complicated fuzzy inference model for the target domain using transfer learning, which took far less time than simply creating it from scratch. However, expressing the link between a complex fuzzy rule's amplitude and phase components in vector form presents difficulties. The rule adaptation phase also requires a flexible and convenient representation for rule adjustment. This is a motivation of the authors to give a visual representation of the relationship between the amplitude and phase part of the complex fuzzy rule (CFR) in this work. In this study, we propose a method to represent CFRs based on CFRG structures that concern both amplitude and phase parts. Then, a novel CFRG-based complex fuzzy transfer learning (denoted RTrieCFTL) is proposed to overcome the limitation of computational time in previous CFTL. The main contributions in this paper can be highlighted as follows:

- Propose a CFRG structure that represents the relationship between amplitude and phase parts by complex fuzzy nodes.
- Propose some algorithms on the CFRG, aiming to prove the efficiency of the proposed theory.
- Introduce an improvement of the fuzzy transfer learning system model (RTrieCFTL) based on the CFRG structure.
- Demonstrate the usefulness of our suggested approach through experiments performed on real decision-making data sets and the UCI regarding reference capability, accuracy, the number of rules, and computational time.

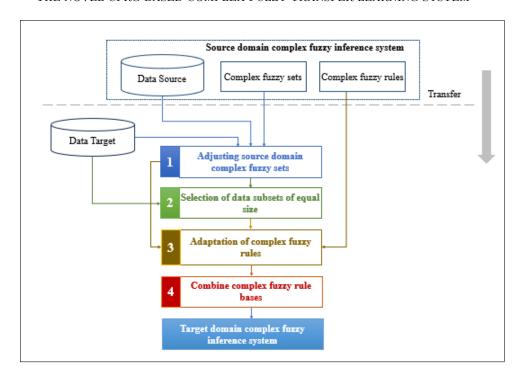


Figure 1: Complex fuzzy transfer learning model (CFTL) [15]

The paper is organized as follows: Section 2 provides the preliminaries, which include the fundamental definitions of the M-CFIS model, CFTL, and tree. Section 3 investigates the novel CFRG-based complex fuzzy transfer learning. Section 4 evaluates the proposed RTrieCFTL by running it on both real-life and benchmark data sets. To conclude, the last part must outline this research's future work.

2. PRELIMINARIES

This section describes basic definitions of M-CFIS, CFTL, and the decision tree structure. These definitions are considered the background knowledge used throughout this work.

2.1. Mamdani complex fuzzy inference system

The M-CFIS is introduced by selvachandran et al. [12] for handling data that have periodic and vague phenomena M-CFIS consists of the following steps.

Let $l_1, l_2, ..., l_m \in \mathbb{C}$ be the data inputs of the fuzzy inference system.

Step 1: Generating the rulebase set of complex fuzzy rules (CFRs) from the input data.

Step 2: Fuzzification, using the Gaussian fuzzy function to fuzzify the input data

$$\operatorname{Re}\left(c\operatorname{Gaussian}\left(l,c,\sigma\right)\right) = \exp\left[-0.5\left(\frac{l-c}{\sigma}\right)^{2}\right],$$

$$\operatorname{Im}\left(c\operatorname{Gaussian}\left(l,c,\sigma\right)\right) = -\exp\left[-0.5\left(\frac{l-c}{\sigma}\right)^{2}\right] \times \left(\frac{l-c}{\sigma^{2}}\right),$$
(1)

with c, σ are the mean and variance of the Gaussian function, and $l \in U$ is an element of U.

- Step 3: Determine the firing strength of a CFR by combining the fuzzified inputs corresponding to CFRs.
- Step 4: Compute the CFR's results by combining the CFR's rule strength and the output membership function using the Mamdani implication.
 - Step 5: Aggregate the CFR's outcomes and give the final complex fuzzy result.
 - Step 6: Defuzzification and return the final crisp output.

2.2. The complex fuzzy transfer learning model

Huong et al. [15] first proposed CFTL, which combines complex fuzzy inference systems (CFIS) and transfer learning. The CFTL framework is described in detail in Figure 1. Four key phases make up the CFTL design. In order to fit the target domain, source data are first changed using a domain adjustment approach. Next, a method for selecting subsets based on the target's output labels and attribute fields is shown. Each subgroup will then generate a set of complex adaptive fuzzy rules (CFRs) as the data records are changed to change the rules. After combining adaptive CFRs, the final adaptable is produced.

The CFTL model is proposed for the transfer learning process on complex fuzzy inference systems where the source and target domains are the same in the number of attributes, similar in the task but different in distribution. The CFTL transfer learning model has been applied to build an inference system in a large target data domain by considering a part of the target domain data as the source domain.

2.3. DAG- Directed acyclic graph

A directed acyclic graph (DAG) is formed by vertices and by edges connecting pairs of vertices, where the vertices can be any kind of object that is connected in pairs by edges. In DAG, each edge has an orientation, from one vertex to another vertex. A path in a directed graph is a sequence of edges having the property that the ending vertex of each edge in the sequence is the same as the starting vertex of the next edge in the sequence; a path forms a cycle if the starting vertex of its first edge equals the ending vertex of its last edge. A directed acyclic graph is a directed graph that has no cycles.

2.4. Tree structure

An accessible way to represent and arrange data is through a tree data structure, which is a hierarchical structure. It consists of a group of nodes connected by hierarchically arranged edges that are placed hierarchically.

The root and child nodes are the terms used to describe a tree's highest and lowermost nodes of a tree, respectively. Each node may have several child nodes, and these child nodes may have children of their own, creating a recursive structure. As was already said, a tree is among the most logical, simple, and practical data structures for presenting, locating, and changing related data. Fuzzy decision trees are used to tackle theoretical and real-world machine-learning challenges. They have been the subject of several related studies [4, 16, 17, 18, 19]. The research results demonstrate the advantages of tree structure for modeling machine learning solutions.

3. CFRG - BASED COMPLEX FUZZY TRANSFER LEARNING MODEL

It is clear that because the domain-shifting technique is relatively straightforward, the CFTL model's capacity to adapt rules still needs to be strengthened. In this section, a novel CFRG structure (a tree-like structure) is proposed for representing the CFTL model's rule base in an effort to enhance its performance. This representation of CFRs helps to improve the rule editing process within the CFTL system. Additionally, the rule base is optimized, and the likelihood of numerous duplicate rules is reduced thanks to the representation of the rule using CFRG.

The following are some definitions and algorithms on CFRG.

3.1. The concept of complex fuzzy rule graph (CFRG)

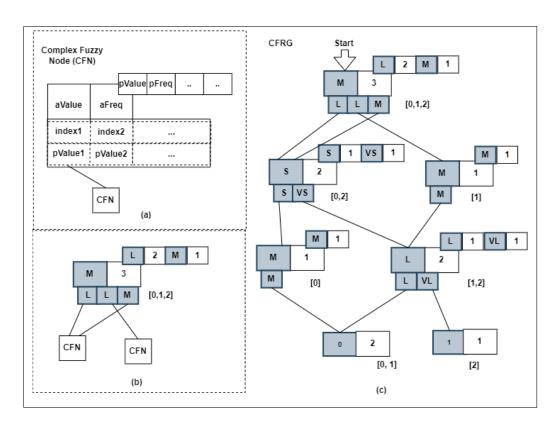


Figure 2: Complex fuzzy node (a, b) and CFRG structure (c)

Definition 1. A complex fuzzy node is an entity that has the structure $CFN(p_A, p_P, p_l)$ with each component in the CFN satisfies the following properties:

 p_A that is list of pairs (aValue, afreq), with aValue is a linguistic variable and freq is its frequency in amplitude element.

 p_P that is a list of pairs (pValue, pfreq), with pValue is a linguistic variable and pfreq is its frequency in phase element.

 p_l is a list of links to connect to child nodes in order of rule and phase term.

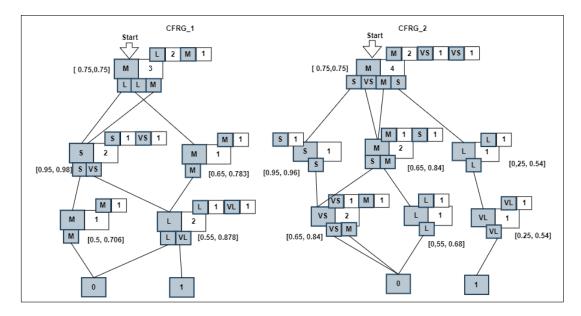


Figure 3: Similarity measure between two CFRGs [15]

Figure 2(a), 2(b) corresponding to describes the CFN according to the Definition 1 and its example. It consists of three components: (M,3) is the first part; a list of pairs (L,2),(M,1) is the second part; and the list of connections to child nodes based on the phase element [L,L,M] with the corresponding index [0,1,2].

The above concept of complex fuzzy node (CFN) defines CFRG as follows.

Definition 2. A complex fuzzy rule tree is a tree structure that represents a set of complex fuzzy rules if these conditions are satisfied:

- Each node in the tree is a CFN that represents the properties of one or more rules that have the same linguistic variable in amplitude term but may differ in phase term.
- Each edge represents a T-norm operator (AND or OR).
- The phase parts of the node that belong to the same rule will have the same index.

According to the definition of a CFRG, a leaf CFN is a node that has no children, or in this case a node is a node with none value in phase part and a child list.

A set of complex fuzzy rules will create n CFRGs, where n is the number of linguistic variables of each attribute in the rule. To better understand CFRG, Figure 2 (c) introduces an example of a CFRG with three complex fuzzy rules as follows. Figure 2 (c) illustrates a CFRG with three complex fuzzy rules as follows:

CFR₁: If $(Ax_1 \text{ is } M \text{ and } Px_1 \text{ is } L)$ and $(Ax_2 \text{ is } S \text{ and } Px_2 \text{ is } S)$ and $(Ax_3 \text{ is } M \text{ and } Px_3 \text{ is } M)$ then 0.

CFR₂: If $(Ax_1 \text{ is } M \text{ and } Px_1 \text{ is } L)$ and $(Ax_2 \text{ is } M \text{ and } Px_2 \text{ is } M)$ and $(Ax_3 \text{ is } L \text{ and } Px_3 \text{ is } L)$ then 0.

CFR₃: If $(Ax_1 \text{ is } M \text{ and } Px_1 \text{ is } M)$ and $(Ax_2 \text{ is } S \text{ and } Px_2 \text{ is } VS)$ and $(Ax_3 \text{ is } L \text{ and } Px_3 \text{ is } VL)$ then 1.

Definition 3. Let CFRG_1 and CFRG_2 be two CFRGs that separated from a parent CFRG. The similarity measures between the CFRG_1 and CFRG_2 are calculated as follows

$$DI = \sum_{i} \frac{DI(CFRG_{-1}, CFRG_{-2})_{i}}{h \times (n(CFRG_{-1}.CFN) + n(CFRG_{-2}.CFN))_{i}},$$
(2)

where, $DI(CFRG_1, CFRG_2)_i = \Sigma_j | CFRG_1.CFN_j.afreq - CFRG_2.CFN_j.afreq|_i$, $i \in [1, h)$, h is the largest level of the structure, CFN_j is the linguistic label of the node, and $n(CFRG_1.CFN)_i$ is sum of afreq of nodes at level i.

Given the CFRG_1 and CFRG_2 in Figure 3. The difference between the CFRG_1 and CFRG_2 is calculated due to Definition 3 as follows:

- At the first level, we have $DI_0 = \frac{|CFRG_1.M.3 CFRG_2.M.4|}{CFRG_1.M.3 + CFRG_2.M.4|} = \frac{1}{7}$.
- At the second level $DI_1 = \frac{|CFRG_1.S.2 CFRG_2.S.1| + |CFRG_1.M.1 CFRG_2.M.2| + |CFRG_1.L.0 CFRG_2.L.1|}{CFRG_1.S.2 + CFRG_1.M.1 + CFRG_2.S.1 + CFRG_2.M.2 + CFRG_2.L.1} = \frac{3}{7}.$
- At the three level $DI_2 = \frac{5}{7}$.
- At the fourth level $DI_3 = \frac{1}{7}$.

The measure of the difference between CFRGs is specified as follows

$$DI = \frac{1}{4} \times (DI_0 + DI_1 + DI_2 + DI_3) = \frac{1}{4} \times (\frac{1}{7} + \frac{3}{7} + \frac{5}{7} + \frac{1}{7}) = \frac{10}{28}.$$

Definition 4. The strength of the node is denoted by (S_{CFN_k}) , is calculated by the following formula

$$S_{CFN_k} = \sqrt{\left(SA_{CFN_k} \cdot \cos(SP_{CFN_k})\right)^2 + \left(SA_{CFN_k} \cdot \sin(SP_{CFN_k})\right)^2} \tag{3}$$

with

$$SA_{CFN_k} = \mu \left(CFN_k.aValue \right) + \left(1 - \mu \left(CFN_k.aValue \right)^{\frac{CFN_k.afreq}{n(CFN_k)}} \right), \tag{4}$$

$$SP_{CFN_k} = \max_{i=1,m} \left(\omega \left(CFN_k.pValue_i \right) + \left(1 - \omega \left(CFN_k.pValue_i \right) \frac{CFN_k.pValue_i.afreq}{CFN_k.afreq} \right) \right), \quad (5)$$

where,

- SA_{CFN_k} and SP_{CFN_k} are the strengths of the amplitude and phase, respectively.
- $\mu(CFN_k.aValue)$, $\omega(CFN_k.pValue_i)$ are the amplitude and phase of each complex fuzzy node, respectively.
- m the number of phase of the node.
- $n(CFN_k)$ the number of nodes at level k.

For example, given a node CFN with an amplitude value of 0.65, the node's frequency is 4; The corresponding list of phase and frequency values is (0.02, 1), (0.05, 3); The number of nodes at the level is 10. The node strength is calculated according to Definition 4 as follows

$$SA_{CFN} = 0.65 + \left(1 - 0.65^{\frac{4}{10}}\right),$$

 $SP_{CFN} = 0.05 + \left(1 - 0.05^{\frac{3}{4}}\right),$
 $S_{CFN} \approx 0.808.$

Definition 5. Given a CFRG, the strength of the CFRG is calculated as follows

$$S_{CFRG} = \prod_{1,h-1} \max_{1,n_k} (S_{CFN_k}), \tag{6}$$

with S_{CFN_k} is the strength of the node at the k^{th} level, n_k is the number of nodes at the k^{th} level, h is the number of attributes of the complex fuzzy rule.

3.2. A novel CFRG structure for complex fuzzy transfer learning model

As the above review about the CFTL [15], the limitation of CFTL is that it is time-consuming. One of these reasons that led to this limitation is in the rule adapting process. To solve this problem, we suggest introducing a novel representation of the fuzzy rule in CFTL using the CFRG structure (as shown in Figure 4).

Compared with the previous model, we have added some special stages as follows: Firstly, the process of establishing/constructing the parent CFRG from the rulebase in the source domain. Then, with the aim to reduce the complexity of the parent CFRG, we execute to divide the parent CFRG into some child CFRGs. After separating the parent CFRG, the process of modifying and refining rules and updates on each CFRG. Finally, the process of merging the CFRGs from the set of child/sub CFRGs is executed. Details of algorithms are described in the appendix.

3.2.1. Process of establishing the CFRG

The number of linguistic variables in the first attribute of the complex fuzzy rulebase from the source domain determines the number of CFRG generated. Each CFRG will represent rules whose first attribute has the same value. Hence, the first attribute helps to determine which rule should be added to the CFRG.

3.2.2. Extract a CFRG child

As mentioned above, the process of adjusting the source domain to match the data in the target domain in the CFTL model has increased the system's computational time. One of the reasons is that the system must go through all the source data to adjust it to match the target data. To reduce the time of this source domain adjustment process, we divide the data in this source domain into many subsets. Each time the adjustment process executes to modify the source data compared with the target domain, only these subsets are used. Also, with the idea of reducing the system's computing time when browsing all the data in the complex fuzzy rule tree, a method of splitting the parent tree into many subtrees is proposed.

3.2.3. Rule adaptation on CFRGs

The rule adaptation process on the CFRG has many different adjustments compared to the rule adaptation process in the previous CFTL model. For CFRG, breadth-first tree traversal is used to highlight the fire rules in the rule system. The candidate rules are still determined similarly to the CFTL model. In addition, we propose to use the phase component

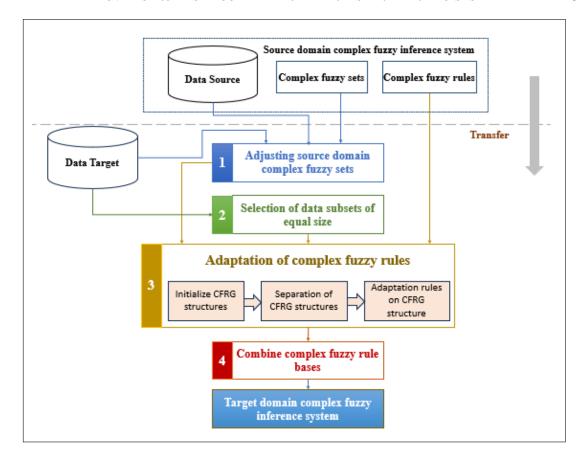


Figure 4: A novel CFRG structure for complex fuzzy transfer learning model (CFRGTL)

value at each complex fuzzy node to suggest rule editing during the rule adaptation process. This helps reduce the calculation time of the CFTL model during the rule adaptation process.

3.2.4. Merge complex fuzzy rule trees

After editing and adapting the rules are completed, those rules will be merged and added to the original CFRG. This process is repeated for all subtrees. Finally, a new CFRG representing the set of CFRs for the target domain data will be obtained. It is said that the representation of the rulebase based on CFRG gives a new approach to refining CFRs.

3.3. Alogrithms on CFRG

Suppose there is a complex fuzzy rule tree as depicted in Figure 1. In a CFRG, each node will determine a corresponding level of that node. Thanks to the list of nodes arranged by class, it helps access nodes quickly. Some algorithms used on CFRG include: adding a CFN to the CFRG, traversal rules in depth, traversal rules by breadth, complex fuzzy inference o CFRG, remove a rule from the CFRG, extract a CFRG child from a CFRG parent, search a rule on the CFRG, merger two CFRGs, editing a CFN on CFRG. They are represented clearly in the appendix.

4. EXPERIMENTAL RESULTS

This section aims to evaluate the performance of CFRG in some decision-making support problems as follows.

4.1. Experimental environtment

This section includes a performance comparison with a complex fuzzy rule-based system [13]. For Python operation, they ran the models on a Lenovo laptop with a Core i7 processor. UCI medical datasets and real-world medical datasets with two label outputs make up the experimental datasets used in the evaluation. There are three datasets from the UCI Machine Learning Repository [20] (Breast Cancer, Diabetes, and Credit Card) as well as actual medical data (Liver illness) from Thainguyen Hospital. The detail of experimental datasets is visually described in Table 1.

No. Dataset No. of instance No.of attributes No. of label Diabetes 390 5 $\overline{2}$ Breast-Cancer (WBCD) 683 9 $\overline{2}$ 3 Credit Card 8636 16 7 Liver 4156 11 2 4

Table 1: Datasets summary

Table 2: Compared results of the inference capability

No.	Dataset	No. of in-	No.for cre-	No. for	Time of M-	Time of
		stance	ating rules	testing	CFIS	CFRG
1	Diabetes	390	312	78	0.1099	0.0786
2	Breast-Cancer (WBCD)	683	546	137	0.7483	0.5821
3	Credit Card	8636	6909	1727	240.223	126.243
4	Liver	4156	3325	831	23.4976	11.7312

Two scenarios have been designed where Scenario 1 is used to evaluate the inference capability of the algorithm. Scenario 2 aims to compare the performance capability of the model in terms of accuracy, number of rules, and computational time.

4.2. Experimental results

To demonstrate the fast inference ability of the proposed CFRG, for each data set in Table 1, an 8:2 ratio is divided, in which 80% of the data is used to generate complex fuzzy rules, the remaining 20% is used to test inference ability. Comparison results of inference ability between the proposed CFRG and M-CFIS models are presented in Table 2.

The experimental results in Table 2 show that using the rule tree structure allows performing the inference process faster than organizing rules on a conventional two-dimensional array as in MCFIS. With two small-sized data sets, Diabetes and breast cancer, the inference time on CFRG is about 27%-28% faster than conventional MCFIS. However, when performing inference on the two data sets Credit Card and Liver with larger sizes in both the number of records and the number of attributes, the inference time was reduced quite a lot, about 45%-50%. This shows that CFRG-based inference is quite effective for large datasets.

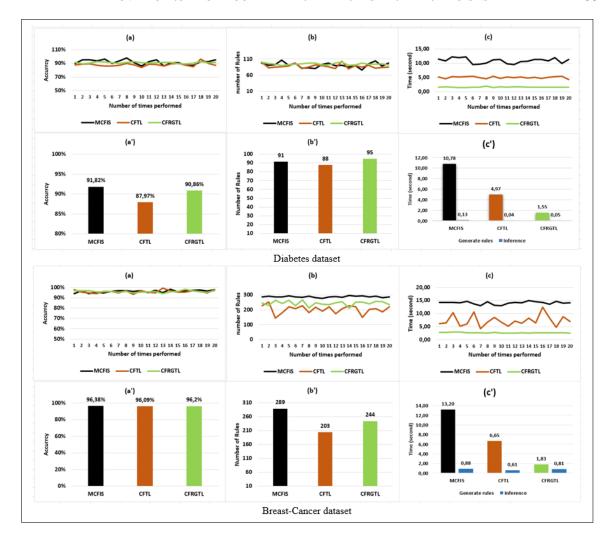


Figure 5: Results on the Diabetes dataset and Breast-Cancer dataset

(a) Accuracy; (a') Average accuracy;

(b) Number of rules; (b') Average number of rules;

(c) Time; (c') Average time.

The second experimental scenario aims to evaluate the effectiveness of the learning capacity of the transfer learning system when solving real problems with a small number of rules and little data in the training data. The CFS rule-generated mechanism in MCFIS is employed for full training data in the first situation but only 30% of training data in the second. In the second scenario (the scenario for CFTL and CFRGTL), 30% of the training data is divided into two parts: 10% of the training data is utilized to generate rules using the MCFIS mechanism, and the remaining 20% is used as the Dsub dataset. Experimental comparison of two complex fuzzy rule representation models evaluated on three indicators of calculation time, accuracy, and number of rules. The experimental scenario is also divided as in the CFTL model to evaluate the effectiveness of inference on the two rule bases.

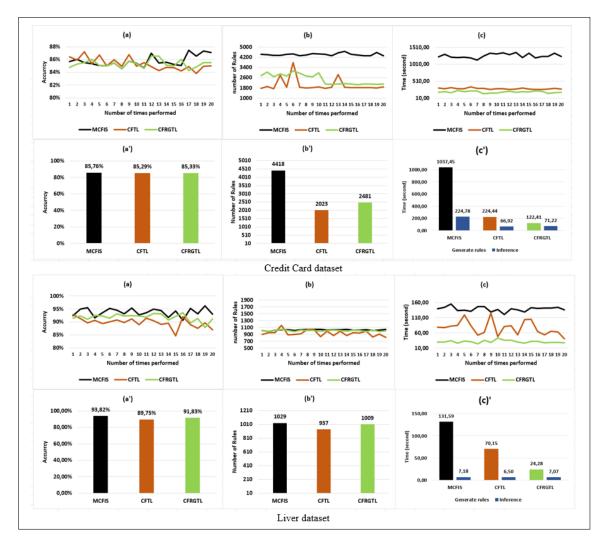


Figure 6: Results on the Credit Card dataset and Liver dataset: (a) Accuracy; (a') Average accuracy; (b) Number of rules; (b') Average number of rules; (c) Time; (c') Average time.

After carrying out the experiments, the obtained results of four datasets are presented in Figures 5, 6, respectively.

Experimental results on all datasets show that the new CFRGTL model has an accuracy equivalent to the MCFIS model but slightly higher than CFTL. However, the variance of the measured accuracy values across trials is smaller than that of the CFTL method (Figure 5 (a')). This proves the stability of the inference of the new model. In addition, the results on the data sets also clearly show that the calculation time on all 4 data sets of the CFRGTL model is faster than the CFTL and much faster than the MCFIS. This can be explained by the representation of the rule system based on the tree structure and the parallel processing of splitting subtrees to search and adjust the rules. However, the number of rules generated in CFRGTL is more than that in CFTL, this is the reason why the inference time on the Test data set is higher. Although the total time of rule generation and inference stage on the CFRGTL model is smaller. The number of rules generated at each learning session in the CFRGTL model is more stable than in the CFTL.

5. CONCLUSIONS

A novel way to demonstrate the frequency factor of rule characteristics is by using a CFRG to describe complex fuzzy rules. The simple representation employs multithreaded, quick inference methods. This has made it quicker to adjust the rule basis for the complicated fuzzy transfer learning system that was previously presented. Furthermore, the intricate fuzzy tree structure encouraged the choice of related rule subsets, which encouraged lowering variance for randomized trials. We may develop new rules with a variety of different features on the tree for future studies using the rule tree structure, which will allow us to extend new branches based on the original tree in a way that has greater explanatory properties.

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