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REVIEW

ATTRIBUTE REDUCTION BASED ON ROUGH SET THEORY AND ITS EXTENSIONS: A REVIEW

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Abstract. In the face of the explosive growth in data volume, current technologies have encountered many difficulties in both storage and knowledge discovery processes. Moreover, the quality of data has also deteriorated due to excessive noisy information, which reduces the effectiveness of machine learning models. Therefore, many solutions have been proposed, in which attribute reduction has emerged as an important research direction. Currently, research on attribute reduction has become very active and is primarily focused on the processing of decision tables. In this area, research on attribute reduction based on rough set theory and its extensions is considered a promising direction, which has been yielding many impressive results. To gain a clearer understanding of the attribute reduction research direction, this study will provide an overview of the methods of attribute reduction from their inception to the methods proposed in recent times.

Keyword. The reduct, rough sets, fuzzy rough sets, intuitionistic fuzzy rough sets.

1. INTRODUCTION

In addition to the basic tasks of data preprocessing, attribute reduction has the main function of reducing the dimensionality of the data while preserving the information compared to the original dataset. Additionally, attribute reduction improves the effectiveness of data mining algorithms by: increasing the simplicity and understandability of rules, enhancing the performance of algorithms by eliminating redundant attributes, and increasing the accuracy of models by removing noisy attributes. Research on attribute reduction is rapidly growing and primarily focuses on processing decision tables. Currently, there are

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many approaches to the attribute reduction problem. Among them, approaches such as filtering, wrapper, and hybrid can be mentioned. Each approach has its own objectives, either to reduce the number of attributes or to improve the accuracy of the classification model. However, studies often use the filtering approach due to its ability to find a representative reduct that performs well on classification models. The methods following the filtering approach currently all use a measure as the basis to evaluate the significance of a obtained attribute subset. Through the properties of the measure, this method defines a reduct as a subset of attributes that preserves the amount of information in the data compared to the entire original attribute set. In this way, the most significant attributes based on the measure will be selected consecutively until the stopping condition occurs or the obtained attribute subset satisfies the properties of the reduct. Finally, the method will use the obtained reduct to evaluate performance on learning algorithms.

Based on the approaches presented, the attribute reduction problem has been implemented on various models. These models always have distinct features and are applied to attribute reduction algorithms to handle specific cases of decision tables. However, in addition to their advantages, they also have limitations when applied in practice. A typical example is the rough set model proposed by Pawlak in 1984 [1]. This model was the first tool applied to the attribute reduction problem due to its usefulness in solving classification problems and discovering rules when the data contains vague and uncertain information. To provide a comprehensive overview of attribute reduction methods across different models, this paper will thoroughly analyze the advantages and disadvantages of each model, thereby guiding future research directions. Specifically, Section 2 will provide an overview of the attribute reduction problem based on the rough sets and relational database approach. In Sections 3 and 4, we will analyze some attribute reduction algorithms based on the neighborhood rough set model and the fuzzy rough set model. These models are considered as the initial extensions of the rough set model. Section 5 and 6 will present attribute reduction methods based on newly developed models in recent years, such as the intuitionistic fuzzy rough set model and the α, β-level intuitionistic fuzzy rough set model. Finally, Section 7 will present the conclusion on the contributions of the research.

2. ROUGH SETS AND THE RELATIONAL DATABASE APPROACH

First, the research object of rough sets theory is called the decision table, which can be expressed as a pair $\mathcal{I} = (U, C \cup D)$, in which U is a finite nonempty set of objects, C is the set of condition attributes, D is the decision attribute set, satisfying $C \cap D = \emptyset$. Each attribute $b \in C \cup D$ determines a mapping $b : U \to V_b$, where V_b is the value set of attribute b. Then, for $u \in U$, the value of b for u is written as $b\,(u)$.

Without loss of generality, assume that D contains only one decision attribute d (in cases where D has more than one attribute, it can be transformed into a single attribute through encoding). Thus, we only need to consider $\mathcal{I} = (U, C \cup D)$.

In this model, an indiscernibility relation generates equivalence classes, which are used to construct the dependency function between condition and decision attributes [2]. Then, the dependency function is used to determine the classification ability of each attribute in the decision table. The primary advantage of rough set theory is that it does not require any prior or supplementary information about the data, such as probability in statistics, basic probability assignment in Dempster-Shafer theory [3]. Based on this model, several attribute reduction algorithms have been developed, leading to significant success in complete decision tables. Subsequently, M. Kryszkiewicz [4] introduced the concept of a tolerance relation to address issues associated with to incomplete decision tables. Building on tolerance relations and Boolean reasoning methods, the authors in [5] proposed a technique for identifying reducts in incomplete decision tables. Using the relational data model approach [6, 7], the authors in [8] proposed an algorithm to find all reducts in a consistent decision table, and [9] proposed an improved entropy measure to define a new reduct that is equivalent to Pawlak's reduct, serving as the foundation for designing an attribute reduction algorithm on inconsistent information systems. Besides, [10] extended the Liang entropy measure and proposed a heuristic algorithm to find a reduct of the table. Additionally, the authors in [11] demonstrated that reducts in an incomplete information system are equivalent to a Sperner system. In other words, the set of reducts in an incomplete decision table forms a Sperner system, and conversely, for a given Sperner system $\mathcal{K}$, there always exists an incomplete decision table such that its set of reducts is exactly $\mathcal{K}$.

However, these methods are often suitable only for nominal/categorical attributes, while data in practice may comprise continuous numerical/real attributes. To handle this case, traditional methods require a preprocessing step to discretize the data. Naturally, the data discretization process may result in information loss, ultimately leading to a decrease in the classification performance of the obtained reducts.

3. NEIGHBORHOOD ROUGH SETS-BASED ATTRIBUTE REDUCTION

Given a decision table $\mathcal{I} = (U, C \cup D)$, an attribute subset $B \subseteq C$ and two objects $u, v \in U$, the distance between u and v with respect to B, denoted $\delta_B(u,v)$, is determined by

$$\delta_B(u,v) = \sqrt[p]{\sum_{b \in B} |b(u) - b(v)|^p}, \tag{1}$$

where is called Manhattan distance if $p = 1$, Euclidean distance if $p = 2$, and Chebyshev distance if $p = \infty$.

To solve the issue of attribute reduction without the need for discretizing the data, the neighborhood rough set model uses a neighborhood relation instead of the indiscernibility relation in the classical model. Accordingly, each object in the universe is characterized by a neighborhood class, which contains objects that have a neighborhood relation with the given object within a λ radius.

Suppose that λ is a neighborhood radius with a value in the range $[0,1]$, and δ is a given distance function. Then, a binary relation R_B^λ becomes a neighborhood relation on U from the attribute set B is defined as

$$R_B^\lambda = \left\{(u,v) \in U \times U \,\middle|\, \forall b \in B, \delta_{\{b\}}(u,v) \leq \lambda\right\}. \tag{2}$$

Based on the neighborhood relation, the neighborhood class of an object $u \in U$ on the attribute subset B is represented by the following formula

$$[u]_B^\lambda = \left\{v \in U \,\middle|\, (u,v) \in R_B^\lambda\right\}. \tag{3}$$

Clearly, the neighborhood class of any object $u \in U$ is a non-empty set that satisfies $[u]_B^\lambda \subseteq U$. If we consider all the objects in the space U, we obtain a family of neighborhood classes, denoted as $U/R_B^\lambda = \left\{[u]_B^\lambda \,|u \in U\right\}$, which is called a neighborhood cover of the attribute set B. Through these neighborhood classes, the basic concepts of neighborhood rough sets are defined as follows.

Consider a decision table $\mathcal{I} = (U, C \cup D)$, a partition from the decision attribute set $U/D = \{D_1, D_2, \ldots, D_q\}$ and a neighborhood relation R_B^λ, the lower and upper neighborhood approximations of a decision class $D_i \in U/D$ are respectively determined by

$$\underline{\mathrm{N}_B}\left(D_i\right) = \left\{u \left| [u]_B^\lambda \subseteq D_i, u \in U\right.\right\} \tag{4}$$

and

$$\overline{\mathrm{N}_B}\left(D_i\right) = \left\{u \left| [u]_B^\lambda \cap D_i \neq \emptyset, u \in U\right.\right\}. \tag{5}$$

Based on this model, many attribute reduction methods have been proposed. First, Hu et. al. proposed several attribute reduction methods for mixed decision tables (containing both discrete and continuous attributes) with various measures, such as dependency degree [12, 13] and the decision error rate in the neighborhood [14]. In [12], the dependency function has been extended by Hu on the neighborhood rough set model as follows

$$\mathcal{DF}_B\left(D\right) = \frac{|\mathrm{POS}_B(D)|}{|U|}, \tag{6}$$

where $\mathrm{POS}_B(D) = \bigcup\limits_{D_i \in U/D} \underline{\mathrm{N}_B}\left(D_i\right)$ is called neighborhood positive region of B. From this measure, the authors designed the NFARNRS algorithm with the steps presented in Algorithm 1.

Algorithm 1 NFARNRS [12]

Input: A decision table $\mathcal{I} = (U, C \cup D)$ and a threshold λ
Output: One reduct *red*
1: let $red = \emptyset$
2: **for** $b \in C \backslash red$ **do**
3: compute $\mathcal{DF}_{red \cup \{b\}}\left(D\right) = \frac{\left|\mathrm{POS}_{red \cup \{b\}}(D)\right|}{|U|}$
4: compute the significance $\mathrm{Sig}\left(b, red\right) = \mathcal{DF}_{red \cup \{b\}}\left(D\right) - \mathcal{DF}_{red}\left(D\right)$
5: **end for**
6: find b_0 with maximum value $\mathrm{Sig}\left(b_0, red\right)$
7: **if** $\mathrm{Sig}\left(b_0, red\right) > \theta$ **then**
8: $red \leftarrow red \cup \{b_0\}$
9: goto step 2
10: **end if**
11: **else** return *red*

Next, Wang [15] selected an optimal attribute set by using the neighborhood discriminative index. Additionally, Sun et al. in [16] presented the FSNTDJE algorithm for attribute selection based on the entropy measure combined with tolerance dependency in the neighborhood. In other approaches, Wang in [17] developed an attribute reduction algorithm based on the k-nearest neighborhood rough set, and Yang in [18] also developed an algorithm based on the Pseudo-label neighborhood rough set. For incomplete decision tables, Yuan et al. in [19] defined the tolerance neighborhood relation and neighborhood multigranulation rough sets to design an attribute reduction algorithm (PTSIJE-FS), using the entropy measure. The algorithm was then compared with the DMRA [20] and PDJE-AR [21] algorithms, as shown in Table 1.

Table 1: Classification accuracy of algorithms based on neighborhood rough sets using the obtained reducts in the KNN classifier.

ID	Dataset	Raw	FSNTDJE	PDJE-AR	PTSIJE-FS
1	Credit	0.7046	0.7046	0.8551	0.8652
2	Heart	0.7652	0.7741	0.8037	0.8222
3	Sonar	0.7262	0.6975	0.7240	0.8365
4	Wdbc	0.9124	0.9466	0.9561	0.9561
5	Wine	0.9195	0.9057	0.9494	0.9775
6	Wpbc	0.7475	0.7374	0.7576	0.7626
7	DLBCL	0.8960	0.8050	0.9350	0.9870
8	Leukemia	0.7340	0.8330	0.8750	0.9583
9	Lung	0.9310	0.8920	0.9410	0.9890
10	MLL	0.6528	0.9167	0.8722	0.9722
11	Prostate	0.7820	0.7870	0.8680	0.8824

Overall, methods based on the neighborhood rough set approach are highly effective in handling numeric or mixed decision tables owing to their ability to offer a more comprehensive characterization of an object's attributes compared to traditional rough set theory. Notably that the neighborhood rough set model only focuses on objects within a neighborhood. Accordingly, the model can reduce the computational space and minimize the processing for attribute reduction algorithms. However, the neighborhood relation is still simple and does not fully describe the relationship between objects. In reality, decision tables always contain objects with diverse distributions. Therefore, the relationship between objects also needs to be characterized by different values.

4. FUZZY ROUGH SETS-BASED ATTRIBUTE REDUCTION

By combining traditional rough set theory and fuzzy set theory, the fuzzy rough set theory was proposed by Dübois and Prade [22] as an effective model for handling continuous data. In this model, each given object is represented by a fuzzy equivalence class, where the remaining objects from the universe will belong to this class based on a degree of membership. Specifically, the authors in [22] used equivalence relations to approximate the fuzzy sets. Accordingly, each relation $\widetilde{R}_B$ determined on the attribute subset $B \subseteq C$ is called a fuzzy equivalence relation if it satisfies the following conditions for any $u, v \in U$:

1) Reflexivity: $\widetilde{R}_B(u, u) = 1$,
2) Symmetry: $\widetilde{R}_B(u, v) = \widetilde{R}_B(v, u)$,
3) Sup–min transitivity: $\widetilde{R}_B(u, v) \geq \sup_{t \in U} \left\{ \min \left(\widetilde{R}_B(u, t), \widetilde{R}_B(t, v) \right) \right\}$.

Through fuzzy equivalence relations, each object $u \in U$ will define a fuzzy equivalence class, denoted as $[\widetilde{u}]_B$, which consists of all objects in U presented by similarity degrees. Clearly, the family of all fuzzy equivalence classes, $U/\widetilde{R}_B = \{[\tilde{u}]_B | u \in U\}$, form a fuzzy partition on U.

Based on the fuzzy rough set model, the first study applying it to the web classification problem was proposed by Jensen [23]. In this study, the authors developed a dependency function from the traditional rough set model and proposed an attribute reduction algorithm based on the fuzzy rough set model. Subsequently, Tsang et al. [24] defined reducts of decision tables and proposed a method to find all reducts based on indiscernibility matrices. Based on these indiscernibility matrices, many methods have been proposed to process continuous data, such as [25, 26, 27, 28]. Besides, Dai and Xu in [29] designed an attribute reduction algorithm using the information gain ratio. From a hybrid filter-wrapper approach, Zhang et al. in [30] proposed a two-stage attribute reduction algorithm. In the first stage, candidate reducts are generated, and in the second stage, the best reduct is selected with the highest classification accuracy. Based on the fitting model, Wang et al. [31] proposed an attribute reduction method to minimize misclassification from objects in decision tables. To preserving the invariability of knowledge structure, Zhai et al. in [32] designed an attribute reduction method using Łukasiewicz adjoint operators and hedges. Additionally, some typical methods of fuzzy rough set theory include fuzzy distance [33, 34], fuzzy positive region [35, 36, 37], fuzzy mutual information [38], fuzzy entropy [39, 40], and fuzzy information granulation [41]. In [33], the authors proposed a distance measure called fuzzy partition distance and defined an optimal reduct that preserves the information of the decision table

$$\mathrm{Dis}\left(U/\widetilde{R}_C, U/\widetilde{R}_{C\cup D}\right) = \sum_{u\in U} \frac{(|[\widetilde{u}]_C| - |[\widetilde{u}]_C \cap [\widetilde{u}]_D|)}{|U|^2}. \tag{7}$$

Based on the hybrid filter-wrapper approach, they then designed the FW_FDAR attribute reduction algorithm, which is presented in detail in Algorithm 2. Accordingly, the authors also compare the proposed algorithm with several algorithms based on rough set and fuzzy rough set approaches (as shown in Table 2) to demonstrate the effectiveness of the algorithm.

Table 2: Classification accuracy of algorithms based on fuzzy rough sets using the obtained reducts in the CART classifier.

ID	Dataset	FW_FDAR	RDRAR [42]	GFS [43]
1	Libra	0.546±0.028	0.508±0.028	0.496±0.016
2	WDBC	0.889±0.018	0.852±0.028	0.836±0.016
3	Horse	0.765±0.048	0.706±0.032	0.702±0.026
4	Heart	0.768±0.064	0.726±0.038	0.706±0.025
5	Credit	0.802±0.048	0.764±0.027	0.692±0.026
6	German	0.725±0.026	0.706±0.818	0.695±0.028
7	CMC	0.692±0.012	0.505±0.038	0.504±0.026
8	Waveform	0.785±0.016	0.682±0.015	0.652±0.027

In general, experimental results show that attribute reduction algorithms based on fuzzy rough set theory are more effective than traditional algorithms for decision tables with continuous and numeric attributes. However, the fuzzy rough set model still has the following

Algorithm 2 FW_FDAR [33]

Input: A decision table $\mathcal{I} = (U, C \cup D)$
Output: One reduct red

let $\mathcal{W} \leftarrow \emptyset$, $red \leftarrow \emptyset$, Dis $\left(U/\widetilde{R}_{red}, U/\widetilde{R}_{red \cup D}\right) = 1$
calculate Dis $\left(U/\widetilde{R}_{red}, U/\widetilde{R}_{red \cup D}\right)$
// Filter stage: Finding candidates for reduct
while Dis $\left(U/\widetilde{R}_{red}, U/\widetilde{R}_{red \cup D}\right) \neq$ Dis $\left(U/\widetilde{R}_{C}, U/\widetilde{R}_{C \cup D}\right)$ **do**
    for $b \in C \backslash red$ **do**
        compute Dis $\left(U/\widetilde{R}_{red \cup \{b\}}, U/\widetilde{R}_{red \cup \{b\} \cup D}\right)$
        compute Sig $(b, red) =$ Dis $\left(U/\widetilde{R}_{red}, U/\widetilde{R}_{red \cup D}\right) -$ Dis $\left(U/\widetilde{R}_{red \cup \{b\}}, U/\widetilde{R}_{red \cup \{b\} \cup D}\right)$
    end for
    Select b_0 which satisfies: Sig $(b_0, red) = \max\limits_{b \in C \backslash red} \{\text{Sig}\,(b, red)\}$
    $red \leftarrow red \cup \{b_0\}$
    $\mathcal{W} \leftarrow \mathcal{W} \cup red$
end while
// Wrapper stage: Finding the reduct as the candidate with the highest classification accuracy
for $w \in \mathcal{W}$ **do**
    compute the classification accuracy on w using a classifier with 10-fold
end for
$red = w_{best}$ $\triangleright w_{best} \in \mathcal{W}$ is the attribute subset with the highest classification accuracy.
return red

two limitations:

Firstly, the attribute reduction methods based on the fuzzy rough set approach typically involve constructing evaluation, defining the reduct set according to these measures, and developing heuristic algorithms to find reducts that preserves the defined measures. These measures are computed from the primitive computational elements, which represent the cardinalities of fuzzy equivalence classes or fuzzy information granules. In the fuzzy rough set approach, the quantity of fuzzy information granules is calculated by summing the membership values of all the objects in the decision table. As a result, this calculation becomes redundant, unnecessary, and fails to characteristic the relationships between the objects in the decision table.

Secondly, the attribute reduction method based on fuzzy rough sets is less effective when handling datasets with low classification accuracy and inconsistency [44]. This is because the model still cannot effectively restrict the influence of certain noisy objects. Therefore, they still contribute information values to the measure and affect the ability to select an optimal subset of attributes.

From these difficulties, several state-of-the-art models have been proposed. In the following section, we will present these models through their advantages in applying attribute reduction algorithms.

5. INTUITIONISTIC FUZZY ROUGH SETS-BASED ATTRIBUTE REDUCTION

In recent years, researchers have focused on the approach of calculating the reduct of decision tables based on intuitionistic fuzzy rough sets. The advantage of this model is the necessary addition of the non-membership function component, which helps to adjust the information from certain noisy objects in the data to the correct classification [45]. Therefore, the intuitionistic fuzzy set has the ability to classify objects better than the classical fuzzy set, especially on noisy and inconsistent datasets.

We continue to consider a decision table $\mathcal{I} = (U, C \cup D)$, each attribute subset $B \subseteq C$ will determine a binary intuitionistic fuzzy relation $\ddot{R}_B$ in $U \times U$ as follows [46]

$$\ddot{R}_B(u,v) = \left\{ \left((u,v), \gamma_{\ddot{R}_B}(u,v), \eta_{\ddot{R}_B}(u,v) \right) \middle| (u,v) \in U \times U \right\}, \tag{8}$$

where $\gamma_{\ddot{R}_B}(u,v) \in [0,1]$ and $\eta_{\ddot{R}_B}(u,v) \in [0,1]$ are the similarity and diversity degrees, respectively, which satisfy: $0 \le \gamma_{\ddot{R}_B}(u,v) + \eta_{\ddot{R}_B}(u,v) \le 1$. Then, $\ddot{R}_B$ is called a intuitionistic fuzzy tolerance relation if and only if for any objects $u, v \in U$, $\ddot{R}_B$ satisfies the conditions

1) Reflexivity: $\gamma_{\ddot{R}_B}(u,u) = 1$ and $\eta_{\ddot{R}_B}(u,u) = 0$,

2) Symmetry: $\gamma_{\ddot{R}_B}(u,v) = \gamma_{\ddot{R}_B}(v,u)$ and $\eta_{\ddot{R}_B}(u,v) = \eta_{\ddot{R}_B}(v,u)$.

From this concept, Zhou in [47, 48] redefined lower and upper approximations of $X \subseteq U$

$$\underline{\ddot{R}_B}(X) = \left\{ \left(u, \inf_{v \in U} \max\left(\gamma_X(v), \eta_{\ddot{R}_B}(u,v)\right), \sup_{v \in U} \min\left(\gamma_{\ddot{R}_B}(u,v), \eta_X(v)\right) \right) \middle| u \in U \right\}, \tag{9}$$

$$\overline{\ddot{R}_B}(X) = \left\{ \left(u, \sup_{v \in U} \min\left(\gamma_{\ddot{R}_B}(u,v), \gamma_X(v)\right), \inf_{v \in U} \max\left(\eta_{\ddot{R}_B}(u,v), \eta_X(v)\right) \right) \middle| u \in U \right\}. \tag{10}$$

The pair $\left(\underline{\ddot{R}_B}(X), \overline{\ddot{R}_B}(X)\right)$ is referred to as an intuitionistic fuzzy rough set. Based on this model, several approaches that expand the measures from the fuzzy rough set model have been proposed, as shown in Figure 1.

1) Intuitionistic fuzzy positive region: Similar to the idea of extending the positive region measure from the classical model to the fuzzy rough set model, researchers have focused on the properties of intuitionistic fuzzy sets within the rough set space to construct the intuitionistic fuzzy rough set model. In this model, t-norm, t-conorm operators, and the implication operator in the intuitionistic fuzzy set are customized to construct the upper and lower approximations. Consequently, the intuitionistic fuzzy positive region is the ratio of the cardinality of an intuitionistic fuzzy set to the cardinality of the data set. According to the intuitionistic fuzzy positive region approach, Zhang et al. [49] proposed a general framework for the intuitionistic fuzzy rough set model. Using this framework, researchers can develop the intuitionistic fuzzy rough set model by customizing the operators. Huang et al. [45] introduced a novel framework known as the intuitionistic fuzzy multigranulation rough set. This model combines multigranulation rough sets with intuitionistic fuzzy rough sets. Tiwari et al. [50] applied positive region theory to the intuitionistic fuzzy rough set model, developing an algorithm for searching reducts in the decision table. Additionally, Redman in [51] refined some of Tiwari's proofs in [50, 52] by restructuring the rough set model with a new proposal for the intuitionistic fuzzy decision class to reduce noise in

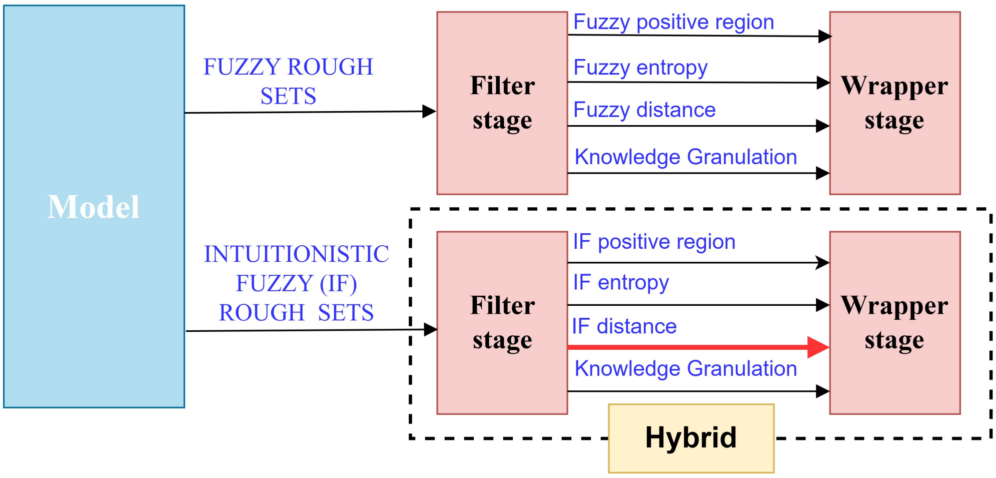

Figure 1: Some measures have been developed based on the intuitionistic fuzzy set model

data sets. Furthermore, Tan in [53] developed a new model based on intuitionistic fuzzy information granules to establish the formula for calculating the intuitionistic fuzzy positive region.

Given a decision table $\mathcal{I} = (U, C \cup D)$, a condition attribute subset $B \subseteq C$ and a partition $U/D = \{D_1, D_2, \ldots, D_l\}$ generated by the decision attribute, for any $(1 \leq i \leq l)$, the lower and upper approximations of decision classes are redefined as follows

$$\underline{\ddot{R}_B}(D_i) = \left\{ \left(u, \inf_{v \notin D_i} \gamma_{\ddot{R}_B}(u,v), \sup_{v \notin D_i} \eta_{\ddot{R}_B}(u,v) \right) \middle| u \in U \right\}, \tag{11}$$

$$\overline{\ddot{R}_B}(D_i) = \left\{ \left(u, \sup_{v \in D_i} \gamma_{\ddot{R}_B}(u,v), \inf_{v \in D_i} \eta_{\ddot{R}_B}(u,v) \right) \middle| u \in U \right\}. \tag{12}$$

Then, the intuitionistic fuzzy dependence function is determined as follows

$$\mathcal{F}(B) = \frac{1}{2} + \frac{1}{2|U|} \sum_{i=1}^{l} \sum_{u \in D_i} \left(\gamma_{\underline{\ddot{R}_B}(D_i)}(u) - \eta_{\underline{\ddot{R}_B}(D_i)}(u) \right). \tag{13}$$

From the proposed measure, the authors in [53] then defined a reduct and a measure for evaluating the significance of an attribute $b \in C$ when added to a subset of condition attributes $B \subseteq C$

$$\mathrm{Sig}(b, B) = \mathcal{F}(B \cup \{b\}) - \mathcal{F}(B). \tag{14}$$

Finally, the authors proposed the IFPR algorithm, as shown in Algorithm 3.

In the experimental process, the authors compared the performance of IFPR with several typical attribute reduction algorithms based on the rough fuzzy set model, such as B-FRFS [25], DM-FR [26], and FA-FPR [37]. Specifically, B-FRFS, DM-FR, and FA-FPR are algorithms based on the fuzzy boundary region, discernibility matrix-based fuzzy rough set, and fuzzy positive region, respectively. The results show that IFPR performs outstandingly by

Algorithm 3 IFPR [53]

Input: A decision table $\mathcal{I} = (U, C \cup D)$.
Output: One reduct red

1: let $red = \emptyset$
2: **for** $b \in C$ **do**
3: compute intuitionistic fuzzy relation $red \cup \{b\}$
4: compute $\gamma_{\underline{\ddot{R}_B}(D_i)}(u)$ and $\eta_{\underline{\ddot{R}_{red}}(D_i)}(u)$ for each $u \in D_i$
5: compute the dependence functions $\mathcal{F}(red)$ and $\mathcal{F}(red \cup \{b\})$
6: compute the significance $\text{Sig}(b, red)$
7: **end for**
8: find b_0 with maximum value $\text{Sig}(b_0, red)$
9: **if** $\text{Sig}(b_0, red) > \theta$ **then**
10: $red \leftarrow red \cup \{b_0\}$ and $C \leftarrow C \setminus \{b_0\}$
11: goto step 2
12: **end if**
13: **else** return red

obtaining an optimal reduct in size and achieving high accuracy on SVM and KNN classification models. Table 3 presents the KNN classification accuracy of IFPR and other algorithms from the obtained reducts.

2) Intuitionistic fuzzy entropy: Entropy in the intuitionistic fuzzy rough set model is better evaluated due to its stricter constraints compared to classical fuzzy rough sets. Following the research direction on entropy measures, Tan et al. [54] developed intuitionistic fuzzy entropy measures and proposed a heuristic algorithm for finding a relative reduct. Additionally, Revanasiddappa [55] applied intuitionistic fuzzy entropy measures for attribute reduction to the text classification problem.

3) Intuitionistic fuzzy distance: In the study [56], the authors developed an intuitionistic fuzzy distance measure and constructed a heuristic algorithm to find a reduct set using a filter-wrapper approach. On the other hand, Anh et al. in [57] extended the formula for calculating the intuitionistic fuzzy partition distance to apply it to decision tables with the addition of object sets. Experimental results show that the reduct set obtained by the intuitionistic fuzzy set has higher classification accuracy compared to the reduct set obtained by the fuzzy rough set on noisy datasets. It can be seen that attribute reduction methods based on rough set and fuzzy rough set models have been developed in a variety of ways. Each method yields very good results in handling specific cases of decision tables. However, these methods are not yet able to effectively handle noisy and inconsistent data. This opens up several research directions to enhance the intuitionistic fuzzy rough set model. Nevertheless, the development of algorithms based on this model also presents some challenges that need to be explored. Firstly, with the addition of non-membership functions, algorithms based on this approach are costly in terms of storage space and have higher computational complexity compared to those based on the fuzzy rough set approach. Secondly, some objects that have a significantly different distribution from the majority of objects in the universe will generate many elements in the intuitionistic fuzzy information granules with small similarity values and large diversity values. Intuitively, these objects are created by noise and affect classifica-

Table 3: Classification accuracy of algorithms based on intuitionistic fuzzy rough sets using the obtained reducts in the KNN classifier

ID	Dataset	IFPR	B-FRFS	DM-FR	FA-FPR	Raw
1	Wine	97.22±2.93	96.04±2.74	95.49±4.41	97.71±2.97	95.49±3.54
2	Heart	85.19±6.30	81.85±6.40	83.33±6.36	74.44±6.16	82.59±6.06
3	Hepatitis	86.50±6.31	85.00±6.63	81.67±7.17	82.83±5.56	80.40±5.84
4	ICU	93.08±2.31	91.21±2.25	91.21±2.25	91.21±2.25	88.61±1.25
5	WPBC	76.63±8.23	73.68±6.87	75.79±5.58	74.71±7.95	76.26±5.89
6	Australian	81.30±5.02	78.27±2.27	79.27±3.18	80.23±5.82	75.25±4.79
7	Soner	75.98±5.43	73.57±5.57	72.14±6.94	74.07±8.94	72.62±7.05
8	Horse	89.13±3.61	91.02±3.67	86.45±5.86	89.68±5.05	88.05±5.58
9	WDBC	96.15±3.04	96.50±2.18	96.50±2.46	96.67±3.00	93.33±2.09
10	Iono	84.12±3.92	82.13±5.56	82.54±4.70	82.72±5.70	82.40±5.02
11	Autovalue_B	91.31±4.57	90.66±4.67	91.35±4.09	91.95±5.09	90.98±4.51
12	Colon	84.58±18.84	87.08±17.17	76.25±12.12	79.58±11.19	73.33±14.05
13	Breast	78.33±11.08	86.67±13.86	90.42±8.84	82.50±10.54	80.42±6.02
14	Leukemia	80.42±6.98	76.19±6.62	84.22±6.46	83.31±8.54	90.98±4.51
15	MLL	98.57±4.52	92.29±10.69	96.73±8.14	97.14±6.02	79.95±15.97
	Average	86.57±6.21	85.54±6.48	85.55±5.90	85.25±6.32	81.73±6.59

tion performance. Clearly, the intuitionistic fuzzy rough set model cannot fully eliminate the impact of these objects. In other words, although adjusted by diversity values, the attribute evaluation measures are still performed on noisy values. Thirdly, correctly classified objects are still adjusted by information from the non-membership function. Therefore, the reduct obtained from algorithms based on the intuitionistic fuzzy set approach may not always be optimal for certain datasets. To address these disadvantages, Anh et al. in [58] defined a new model called the α, β-level intuitionistic fuzzy set. This model not only inherits the advantages of the intuitionistic fuzzy rough set model, but also effectively addresses the disadvantages in some previous models. Accordingly, this study will present the model to highlight these improvements.

6. $\boldsymbol{\alpha, \beta}$-LEVEL INTUITIONISTIC FUZZY SETS-BASED ATTRIBUTE REDUCTION

Firstly, the concept of intuitionistic fuzzy equivalence relation is formed from the intuitionistic fuzzy tolerance relation with the extension of the transitive property. Then, the intuitionistic fuzzy equivalence relation $\ddot{R}_B$ satisfies the properties following

1) Reflexivity: $\gamma_{\ddot{R}_B}(u,u) = 1$ and $\eta_{\ddot{R}_B}(u,u) = 0$,
2) Symmetry: $\gamma_{\ddot{R}_B}(u,v) = \gamma_{\ddot{R}_B}(v,u)$ and $\eta_{\ddot{R}_B}(u,v) = \eta_{\ddot{R}_B}(v,u)$,
3) Sup–min transitivity: $\gamma_{\ddot{R}_{\{c\}}}(u,v) \geq \max\limits_{t\in U}\left\{\min\left(\gamma_{\ddot{R}_{\{c\}}}(u,t), \gamma_{\ddot{R}_{\{c\}}}(t,v)\right)\right\}$,

$$\eta_{\ddot{R}_{\{c\}}}(u,v) \leq \min_{t\in U}\left\{\max\left(\eta_{\ddot{R}_{\{c\}}}(u,t), \eta_{\ddot{R}_{\{c\}}}(t,v)\right)\right\}.$$

It is readily apparent that $\ddot{R}_B$ will generate an intuitionistic fuzzy partition $U/\ddot{R}_B = \{[\ddot{u}]_B | u \in U\}$ on U, where $[\ddot{u}]_B$ is an intuitionistic fuzzy equivalence class of object u ac-

Algorithm 4 IFPD [58]

Input: A decision table $\mathcal{I} = (U, C \cup D)$, levels α, β and a threshold θ
Output: One reduct red

initialize $red = \emptyset$
compute $U/\ddot{R}_D^{\alpha,\beta}$
for $b \in C$ **do**
  if b a is a continous numeric value domain attribute **then**
    compute $U/\ddot{R}_{\{b\}}^{\alpha,\beta}$
  else
    $U/\ddot{R}_{\{b\}}^{\alpha,\beta} = U/\ddot{R}_{\{b\}}$
  end if
end for
compute $U/\ddot{R}_C^{\alpha,\beta} = \bigcap_{b \in C} U/\ddot{R}_{\{b\}}^{\alpha,\beta}$
$red = \{b_0\}$ which satisfies: $\mathcal{D}\left(U/\ddot{R}_{\{b_0\}}^{\alpha,\beta}, U/\ddot{R}\,_{\{b_0\}\cup D}^{\alpha,\beta}\right) = \min\limits_{b \in C}\left\{\mathcal{D}\left(U/\ddot{R}_{\{b\}}^{\alpha,\beta}, U/\ddot{R}_{\{b\}\cup D}^{\alpha,\beta}\right)\right\}$
while $\mathcal{D}\left(U/\ddot{R}_{red}^{\alpha,\beta}, U/\ddot{R}_{red\cup D}^{\alpha,\beta}\right) - \mathcal{D}\left(U/\ddot{R}_C^{\alpha,\beta}, U/\ddot{R}_{C\cup D}^{\alpha,\beta}\right) > \theta$ **do**
  for $b \in C \backslash red$ **do**
    compute $\mathcal{D}\left(U/\ddot{R}_{red\cup\{b\}}^{\alpha,\beta}, U/\ddot{R}_{red\cup\{b\}\cup D}^{\alpha,\beta}\right)$
    compute $\operatorname{Sig}(b, red)$
  end for
  Select b_0 which satisfies: $\operatorname{Sig}(b_0, red) = \max\limits_{b \in C\backslash red}\{\operatorname{Sig}(b, red)\}$
  $red \leftarrow red \cup \{b_0\}$
end while
return red

cording to $\ddot{R}_B$, referred to as an intuitionistic fuzzy information granulation. Next, let α and β be two real numbers in the range [0,1] that satisfy $\alpha + \beta \leq 1$. Then, the ordinary set based on levels α, β of $[\ddot{u}]_B$ is a crisp set and is defined as follows

$$[\ddot{u}]_B^{\{\alpha,\beta\}} = \left\{u \in U \mid \gamma_{[\ddot{u}]_B}(v) \geq \alpha \wedge \eta_{[\ddot{u}]_B}(v) \leq \beta\right\}. \tag{15}$$

It is interesting for us to extend an intuitionistic fuzzy set $[\ddot{u}]_B^{\alpha,\beta}$ based on a combination of each element in $[\ddot{u}]_B^{\{\alpha,\beta\}}$ with the similarity and diversity degrees

$$[\ddot{u}]_B^{\alpha,\beta}(v) = \left(\gamma_{[\ddot{u}]_B^{\alpha,\beta}}(v), \eta_{[\ddot{u}]_B^{\alpha,\beta}}(v)\right) = \begin{cases} [\ddot{u}]_B(v) & \text{if } v \in [\ddot{u}]_B^{\{\alpha,\beta\}} \\ (0,1) & \text{otherwise.} \end{cases} \tag{16}$$

Then, $[\ddot{u}]_B^{\alpha,\beta}$ is an α, β-level intuitionistic fuzzy equivalence class of object u, and a family $\left\{[\ddot{u}]_B^{\alpha,\beta} \middle| u \in U\right\}$ shall create an intuitionistic fuzzy partition on U. This family is called the α, β-level intuitionistic fuzzy partition, denoted by $U/\ddot{R}_B^{\alpha,\beta}$. From this concept, Anh et al. in [58] proposed a formula for calculating the α, β-level intuitionistic fuzzy distance to measure the amount of information in an attribute set B that is not in the decision class, as follows

$$\mathcal{D}\left(U/\ddot{R}_B^{\alpha,\beta}, U/\ddot{R}_{B\cup D}^{\alpha,\beta}\right) = \frac{1}{2|U|}\sum_{u \in U}\sup_{v \in [u]_B^{\{\alpha,\beta\}}}\left\{1 + \gamma_{[\ddot{u}]_B^{\alpha,\beta}}(v) - \eta_{[\ddot{u}]_B^{\alpha,\beta}}(v)\right\}. \tag{17}$$

Through this measure, the authors have redefined a reduct and established the significance of each attribute in the decision table as the foundation for proposing an attribute reduction algorithm based on the filter approach

$$\mathrm{Sig}\,(b, B) = \mathcal{D}\left(U/\ddot{R}_B^{\alpha,\beta}, U/\ddot{R}_{B\cup D}^{\alpha,\beta}\right) - \mathcal{D}\left(U/\ddot{R}_{B\cup\{b\}}^{\alpha,\beta}, U/\ddot{R}_{B\cup\{b\}\cup D}^{\alpha,\beta}\right). \tag{18}$$

Finally, the authors designed the IFPD algorithm to search for an optimal subset of attributes on the fixed decision table. The steps of the method are detailed in Algorithm 4.

To demonstrate the effectiveness of the proposed algorithm (IFPD), the authors then conducted several experiments to compare it with typical algorithms based on the intuitionistic fuzzy rough set model, such as IFPR, IFIE [54], and FMIFRFS [52]. In which, IFPR uses the intuitionistic fuzzy dependency measure, IFIE uses intuitionistic fuzzy entropy, and FMIFRFS uses a fitting model combined with the intuitionistic fuzzy dependency function. The comparison process was conducted on 15 datasets taken from the UCI machine learning repository[1] and evaluated through three criteria: reduct size, execution time (second), and classification accuracy from the obtained reducts.

Table 4: Size of the reduct and execution time (second)

ID	Dataset	IFPD		IFPR		IFIE		FMIFRFS	
		Time	Size	Time	Size	Time	Size	Time	Size
1	Vehicle	0.017	13.7	0.240	12	0.027	6.6	0.091	12.7
2	Satimage	2.692	17.1	5.300	17.2	12.44	18.6	22.70	29.1
3	Ozone	0.017	4.3	0.792	7.1	0.935	12.5	0.392	5.6
4	Qsar	0.058	18	0.548	20.6	0.217	14.4	0.226	23
5	Robot	0.019	6.6	0.579	4.7	0.023	2.8	0.074	5.4
6	Triazines	0.021	12.2	0.665	13.5	0.060	11.8	0.066	12.8
7	Movement	0.071	14.9	7.703	17.6	0.500	39.5	1.064	15.9
8	Sona	0.011	5.8	0.772	21.1	0.087	15.2	0.225	15.6
9	Agnostic	1.885	22.5	4.719	33.3	5.012	31	3.346	30.2
10	Tecator	0.069	11.3	1.087	11.7	0.042	3.7	0.121	7.4
11	LSVT	0.113	12.3	2.744	15.1	0.263	11.3	0.223	9.2
12	PD	1.966	45.1	26.07	44.7	7.752	52.1	7.312	42.7
13	warpAR10P	1.469	20.3	61.72	14.2	7.448	37.5	15.53	18.8
14	Tumors	0.464	2.4	29.58	7	2.868	5.2	0.753	1.3
15	Leukemia	0.726	3.7	29.22	7.1	6.695	11.7	2.311	3.4
	AVERAGE	0.646	14.0	11.45	16.46	2.958	18.26	3.629	15.54

The experimental results in the study have shown (as in Tables 4 and 5) that IFPD outperforms other algorithms in terms of execution time, reduct size, and classification accuracy. This can be demonstrated by two main advantages of the model. The first advantage is based on the nature of the intuitionistic fuzzy rough set model, where the addition of the non-membership function component helps adjust noisy objects to the correct class. The second advantage comes from the characteristics of the α, β-level sets, which help eliminate objects with low similarity or high diversity in the intuitionistic fuzzy information granules.

[1]https://archive.ics.uci.edu/

By removing these objects, the impact of noise information on the attribute evaluation measures is minimized. These two advantages have shown that the algorithms perform very well on datasets with initially low classification accuracy. In other words, these are datasets that contain some objects with distributions differing from most objects in the universe. From these advantages, the authors in [59] continued to use the α,β-level intuitionistic fuzzy set model to develop incremental algorithms for handling practical data scenarios involving the addition and removal of object sets.

Table 5: Comparison on classification accuracies of reduced data with SVM

ID	Dataset	IFPD	IFBR	IFIE	FMIFRFS	Raw
1	Vehicle	$\mathbf{0.763 \pm 0.048}$	0.759 ± 0.052	0.678 ± 0.051	$\mathbf{0.763 \pm 0.043}$	0.752 ± 0.041
2	Satimage	0.896 ± 0.011	0.894 ± 0.013	0.888 ± 0.010	$\mathbf{0.899 \pm 0.015}$	0.900 ± 0.014
3	Ozone	$\mathbf{0.937 \pm 0.013}$	$\mathbf{0.937 \pm 0.013}$	$\mathbf{0.937 \pm 0.013}$	$\mathbf{0.937 \pm 0.013}$	0.937 ± 0.013
4	Qsar	$\mathbf{0.864 \pm 0.025}$	0.862 ± 0.025	0.847 ± 0.034	$\mathbf{0.864 \pm 0.030}$	0.854 ± 0.031
5	Robot	0.452 ± 0.109	0.423 ± 0.084	$\mathbf{0.493 \pm 0.096}$	0.463 ± 0.121	0.397 ± 0.075
6	Triazines	$\mathbf{0.791 \pm 0.078}$	0.785 ± 0.084	0.769 ± 0.100	0.769 ± 0.082	0.764 ± 0.013
7	Movement	$\mathbf{0.792 \pm 0.072}$	0.783 ± 0.077	0.756 ± 0.087	0.781 ± 0.088	0.775 ± 0.077
8	Sona	$\mathbf{0.852 \pm 0.088}$	0.822 ± 0.079	0.822 ± 0.093	0.822 ± 0.089	0.793 ± 0.099
9	Agnostic	$\mathbf{0.827 \pm 0.021}$	0.824 ± 0.018	0.824 ± 0.016	0.825 ± 0.021	0.821 ± 0.018
10	Tecator	$\mathbf{0.942 \pm 0.045}$	0.933 ± 0.049	0.912 ± 0.050	0.925 ± 0.056	0.933 ± 0.053
11	LSVT	0.896 ± 0.124	0.847 ± 0.136	0.872 ± 0.139	$\mathbf{0.913 \pm 0.079}$	0.809 ± 0.122
12	PD	$\mathbf{0.866 \pm 0.034}$	0.849 ± 0.036	0.865 ± 0.036	0.857 ± 0.043	0.857 ± 0.033
13	warpAR10P	0.723 ± 0.146	0.654 ± 0.196	$\mathbf{0.738 \pm 0.224}$	0.654 ± 0.162	0.677 ± 0.148
14	Tumors	$\mathbf{0.667 \pm 0.157}$	0.617 ± 0.113	0.650 ± 0.123	$\mathbf{0.667 \pm 0.157}$	0.650 ± 0.166
15	Leukemia	$\mathbf{0.930 \pm 0.120}$	0.871 ± 0.125	$\mathbf{0.930 \pm 0.100}$	0.914 ± 0.145	0.875 ± 0.100
	AVERAGE	$\mathbf{0.813 \pm 0.07}$	0.791 ± 0.08	0.795 ± 0.08	0.804 ± 0.08	0.786 ± 0.07

In Table 6, we summarize the complexity of several typical algorithms presented in the paper, where we denote n as the number of attributes, m as the number of objects, and T as the processing time of a classification model. All of these algorithms have been proven to have polynomial time complexity and are fully applicable to practical scenarios.

Table 6: Computational complexity of algorithms

ID	Algorithm	Model	Complexity
1	NFARNRS	Neighborhood rough sets	$O(n^2 \log m)$
2	PTSIJE-FS	Neighborhood multigranulation rough sets	$O(n^3 m)$
3	FW_FDAR	Fuzzy rough sets	$O(n^2 m^2) + O(Tn)$
4	IFPR	Intuitionistic fuzzy rough sets	$O(nm^2)$
5	IFPD	α,β-level intuitionistic fuzzy sets	$O(n^2 m^2)$

As the results have shown, the α,β-level intuitionistic fuzzy sets have demonstrated exceptional performance when processing noisy and inconsistent data tables. However, compared to previous models such as rough sets, neighborhood rough sets, and fuzzy rough sets, the processing time of this model has not yet seen significant improvement. With the rapid development of information technology and high-performance computing systems, parallel and multi-threaded processing algorithms can be applied to significantly improve the efficiency of this model. Integrating these algorithms into attribute reduction models will not only help reduce computation time but also optimize system resources, leading to substantial performance improvements, especially when working with large and complex datasets. Furthermore, with the scalability and flexibility of modern computing systems, attribute reduction algorithms based on parallel computing approaches will play a crucial role in optimizing data analysis processes in the future.

7. CONCLUSIONS

Attribute reduction is one of the important problems that has received significant attention from the research community. Over many years of development, attribute reduction methods have continuously evolved to achieve breakthroughs in performance, aiming to effectively address challenges in the context of big data. Through diligent research efforts, researchers have continuously proposed various models to develop attribute reduction methods that can handle different data scenarios. In this study, we have synthesized and analyzed several models, highlighting their advantages and limitations when applied to attribute reduction algorithms. Specifically, the rough set model proves effective when handling uncertain data with discrete attributes, but it encounters difficulties when applied to numerical or continuous data. To overcome this limitation, the rough set neighborhood model and its subsequent extensions were developed. However, these models focus only on objects within a neighborhood, assuming that all objects in a neighborhood have the same classification ability, despite their varying importance. This leads to limitations when applied to data classes with diverse distributions. The fuzzy rough set model was also developed to represent the characteristics of objects in a fuzzy equivalence class through the degree of similarity between objects. However, on noisy data, this model has not yet achieved high performance. To overcome this issue, the intuitionistic fuzzy rough set model was proposed, demonstrating improved performance on noisy data, although the processing time is slower due to the addition of the non-membership function in the model. Therefore, in the future, studies could propose a new model that combines the advantages of the current models to be effectively applied to different data classes in practice. Additionally, the consideration of parallel processing approaches in attribute reduction algorithms should be emphasized, especially as high-performance computing systems continue to evolve.

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